

THE CASIMIR EFFECT FOR SUSY SOLITONS*

ANTON REBHAN

*Institute for Theoretical Physics, Vienna University of Technology,
Wiedner Hauptstr. 8–10, A-1040 Vienna, Austria*

PETER VAN NIEUWENHUIZEN

*C. N. Yang Institute for Theoretical Physics,
SUNY Stony Brook, NY 11794-3840 USA*

ROBERT WIMMER

*Institute for Theoretical Physics, University of Hanover,
Appelstr. 2, D-30167 Hanover, Germany*

We discuss new insights into the quantum physics of solitons developed since 1997: why quantum corrections to the mass M and the central charge Z of solitons in supersymmetric (susy) field theories in 1+1 and 2+1 dimensions are nonvanishing, despite the fact that the zero-point energies of bosons and fermions seem to cancel each other, and the central charge is an integral of a total space derivative which naively seems to get contributions only from regions far removed from the soliton. Crucial are: (1) the requirement that the regularization scheme not only makes calculations finite, but it also should preserve (ordinary) supersymmetry, (2) the renormalization condition that tadpoles vanish in the *trivial* vacuum, (3) an anomaly in the central charge which is actually needed to saturate the Bogomolnyi bound, (4) the influence of the winding of classical fields on the quantum fields far away from the soliton. A new result is announced¹: for the susy vortex solution in 2+1 dimensions, the quantum corrections to M and Z are both nonvanishing, but they continue to saturate the quantum Bogomolnyi bound. Claims in the literature that no “multiplet shortening” arises and hence saturation need not take place, are shown to be incorrect because they are based on the assumption that a second fermionic zero mode exists. We show however that the latter is singular, and thus must be rejected.

1 Introduction

The Casimir effect usually deals with zero point energies in the space between plates, or between a small ball and a plate as we heard at this conference. In certain quantum field theories one has solitons (time-independent nonsingular solutions of the *classical* field equations in Minkowski space with *finite energy*), and the quantum fluctuations around these classical solutions also lead to zero-point energies and thus a Casimir effect. Whereas one might perhaps argue that the Casimir effect is only a dual formulation of the van der Waals forces between the plates, it is much easier to compute the forces using the Casimir

*TO APPEAR IN THE PROCEEDINGS TO THE 6TH WORKSHOP ON QUANTUM FIELD THEORY UNDER THE INFLUENCE OF EXTERNAL CONDITIONS (QFEXT03) NORMAN, OKLAHOMA, USA, 2003/09/15-19

approach. In quantum field theory, one could consider two (or more) solitons, and calculate their attraction due to the Casimir effect, but here we shall be interested in the vacuum energy around one soliton. This is a topic which has an enormous literature. For a review of the situation till the 1980's, see ².

Since the early 1970's when susy was discovered, it is known that in susy field theories the bosonic and fermionic zero point energies *naively* cancel. However, the arguments that the zero point energies should cancel are incomplete when the background is a soliton; in fact, quantum corrections to the soliton mass in supersymmetric field theories are in general nonvanishing^{3,4,5,6,7,8,9}. In addition to the mass M , solitons in supersymmetric theories have another quantum number, their central charge, usually denoted by Z . This central charge is classically a topological quantity: it is given by the space integral of a total space derivative, and hence only depends on the fields at the boundary at infinity. For a long time it was believed that the quantum corrections to Z should vanish because all quantum corrections would have to occur far away from the soliton where physics should be the same as if no soliton were present. It was noticed in 1997 that this would mean the Bogomolnyi bound¹⁰ $M = |Z|$ would be violated⁶. However, although the interactions originating at the soliton can be ignored because they are short range, the topology of these asymptotic fields reflects the presence of a soliton somewhere in the middle. We shall show that nonvanishing quantum corrections to the central charges of soliton may arise either due to a new kind of anomaly^{7,9} (in an odd number of space dimensions), or due to the topology of the fields at infinity, or perhaps due to both effects.

One way to intuitively understand this anomaly is to note that if one uses point splitting to regulate the expression for the central charge at the quantum level, the total derivative ceases to be a total derivative, and its space integral ceases to vanish. Another way, appreciated by people who are familiar with susy, is to note that the well-known conformal-susy anomaly $\gamma \cdot j$ and the well-known trace anomaly T_μ^μ belong to a susy multiplet of anomalies that also contains the central charge anomaly. If one has one of these anomalies, and one does not break ordinary susy, the other anomalies are inevitable⁹.

In susy field theories, the corresponding superalgebra may contain terms with central charges. For example, for the susy $N = 1$ Higgs system in 1+1 dimensions (the susy kink) the superalgebra reads

$$\{Q^+, Q^+\} = H + Z, \quad \{Q^-, Q^-\} = H - Z, \quad \{Q^+, Q^-\} = P, \quad (1)$$

where Q^\pm are the susy generators (real 2-component spinors whose components are denoted by $+$ and $-$), H is the Hamiltonian, and P the translation generator. (One could add the Lorentz generator J_{01} but we shall not need it). The central charge generator Z takes on the following form in terms of the real scalar Higgs field φ

$$Z = \int_{-\infty}^{\infty} dx \partial_x W(\varphi), \quad \partial_x W(\varphi) = U(\varphi) \partial_x \varphi. \quad (2)$$

The potential V is related to U by $V = \frac{1}{2}U^2$.

In 1+1 dimensions we can also construct an $N = 2$ susy model with a kink, but then there are no quantum corrections to M and Z .⁷ In 2+1 dimensions there is an $N = 2$ susy extension of the abelian Maxwell-Higgs model^{11,12} (one can also add a Chern-Simon term to this system^{13,14}) and this model we discuss below. One can truncate this $N = 2$ model down to an $N = 1$ model. And in 3+1 dimensions, $N = 2$ models have susy monopoles but whether M and Z receive quantum corrections in these models is an open question^{15,16} (we are studying these systems).

2 The Nielsen-Olesen vortex with $N = 2$ susy

a. Classical model: We now study well-known Nielsen-Olesen vortex, an abelian Maxwell-Higgs system. We consider the $N = 2$ supersymmetric extension which contains in addition to the abelian $U(1)$ gauge field A_m ($m = 0, 1, 2$) a complex scalar ϕ and a complex 2-component spinor ψ (which form together a $N = 2$ “matter multiplet” in 2+1 dimensions), and further a complex 2-component spinor χ called the gaugino and a real scalar N (the fields A_m, χ, N form the $N = 2$ vector multiplet in 2+1 dimensions). We take the $N = 2$ model instead of the $N = 1$ model because we shall use *dimensional regularization*, and for this scheme to preserve susy we need to find a model in higher dimensions (in 3+1 dimensions) which is susy and which yields upon dimensional reduction our susy vortex in 2+1 dimensions. The simplest case is an $N = 1$ model in 3+1 dimensions, whose action reads in superspace $\mathcal{L} = \int d^2\theta W^\alpha W_\alpha + \int d^4\theta \bar{\phi} e^{-eV} \phi + \kappa \int d^4\theta V$. Dimensional reduction (ignoring dependence on the coordinate z , and decomposing $A_\mu \rightarrow \{A_m, A_3 = N\}$ with $\mu = 0, 1, 2, 3$ but $m = 0, 1, 2$) yields the $N = 2$ model in 2+1 dimensions^a

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \bar{\chi}^{\dot{\alpha}} i\bar{\sigma}_{\dot{\alpha}\beta}^\mu \partial_\mu \chi^\beta + \frac{1}{2}D^2 + (\kappa - e|\phi|^2)D - |D_\mu \phi|^2 + \bar{\psi}^{\dot{\alpha}} i\bar{\sigma}_{\dot{\alpha}\beta}^\mu D_\mu \psi^\beta + |F|^2 + \sqrt{2}e [\phi^* \chi_\alpha \psi^\alpha + \phi \bar{\chi}_{\dot{\alpha}} \bar{\psi}^{\dot{\alpha}}], \quad (3)$$

where $D_\mu = \partial_\mu - ieA_\mu$ when acting on ϕ and ψ , and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Elimination of the auxiliary field D yields the scalar potential $\mathcal{V} = \frac{1}{2}D^2 = \frac{1}{2}e^2(|\phi|^2 - v^2)^2$ with $v^2 \equiv \kappa/e$.

The classical vortex solution is given by

$$\phi_V = e^{in\theta} f(r), \quad eA_+^V = -ie^{i\theta} \frac{a(r) - n}{r}, \quad A_\pm^V \equiv A_1^V \pm iA_2^V \quad (4)$$

^aOur conventions are $\eta^{\mu\nu} = (-1, +1, +1, +1)$, $\chi^\alpha = \epsilon^{\alpha\beta} \chi_\beta$ and $\bar{\chi}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}} \bar{\chi}_{\dot{\beta}}$ with $\epsilon^{\alpha\beta} = \epsilon_{\alpha\beta} = -\epsilon^{\dot{\alpha}\dot{\beta}} = -\epsilon_{\dot{\alpha}\dot{\beta}}$ and $\epsilon^{12} = +1$. In particular we have $\bar{\psi}_{\dot{\alpha}} = (\psi_\alpha)^*$ but $\bar{\psi}^{\dot{\alpha}} = -(\psi^\alpha)^*$. Furthermore, $\bar{\sigma}_{\dot{\alpha}\beta}^\mu = (-\mathbf{1}, \vec{\sigma})$ with the usual representation for the Pauli matrices $\vec{\sigma}$, and $\bar{\sigma}^{\mu\dot{\alpha}\beta} = \sigma^{\mu\beta\dot{\alpha}}$ with $\sigma^{\mu\beta\dot{\alpha}} = (\mathbf{1}, \vec{\sigma})$.

or, alternatively (with $k, l = 1, 2$),

$$\phi_V = \phi_V^1 + i\phi_V^2 = \left(\frac{x^1 + ix^2}{r} \right)^n f(r), \quad eA_k^V = \frac{\epsilon_{kl} x^l}{r} \frac{a(r) - n}{r}, \quad (5)$$

where $f'(r) = \frac{a}{r}f(r)$ and $a'(r) = re^2(f(r)^2 - v^2)$ with boundary conditions

$$\begin{aligned} a(r \rightarrow \infty) &= 0, & f(r \rightarrow \infty) &= v, \\ a(r \rightarrow 0) &= n + O(r^2), & f(r \rightarrow 0) &\rightarrow r^n + O(r^{n+2}). \end{aligned} \quad (6)$$

We shall mainly discuss the solution with $n = 1$.

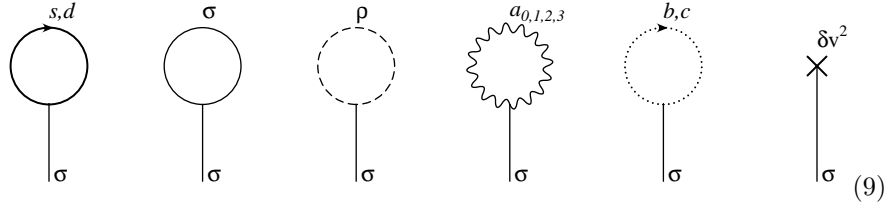
b. Quantization and renormalization: we add a gauge fixing term which diagonalizes the kinetic term (an R_ξ gauge) and from it we obtain in the usual way the corresponding Faddeev-Popov ghost action. Setting $\phi = \phi_V + \eta$ and $A_\mu = A_\mu^V + a_\mu$, the gauge-fixing term is quadratic in quantum fields

$$\mathcal{L}_{\text{g.fix}} = -\frac{1}{2\xi}(\partial_\mu a^\mu - ie\xi(\phi_V \eta^* - \phi_V^* \eta))^2. \quad (7)$$

The corresponding Faddeev-Popov Lagrangian reads

$$\mathcal{L}_{\text{ghost}} = b \left(\partial_\mu^2 - e^2 \xi \{ 2|\phi_V|^2 + \phi_V \eta^* + \phi_V^* \eta \} \right) c. \quad (8)$$

There are no divergences in 2+1 dimensions at the one loop level if we use dimensional regularization, hence one does not need renormalization to make loops finite, but one still must account for *finite renormalization*. As always in quantum field theory, we require that the vacuum expectation value (vev) of quantum fields vanishes, $\langle \eta \rangle = 0$. (If $\langle \eta \rangle \neq 0$, one must add infinitely many trees, and summing these one regains the case that $\langle \eta \rangle = 0$). We have two sectors in the theory: the trivial sector where $\phi = v$ is a solution, and the vortex sector where the vortex solution forms the background. In the trivial sector we set $\phi = v + \eta$, and $\frac{r}{e} \equiv v_0^2 = v^2 + \delta v^2$, and fix δv^2 such that tadpoles vanish in the trivial vacuum:



$$= (-em) \times$$

$$\left\{ -2\text{tr} \mathbf{1}_2 I(m) + \frac{3}{2} I(m) + \frac{1}{2} I(\xi^{\frac{1}{2}} m) + [3I(m) + \xi I(\xi^{\frac{1}{2}} m)] - \xi I(\xi^{\frac{1}{2}} m) - \delta v^2 \right\}.$$

The tadpoles consist of two fermion loops (with $s = (\psi + i\chi)/\sqrt{2}$ and $d = (\psi - i\chi)/\sqrt{2}$), the Higgs scalar σ , the Goldstone boson ρ , the fields a_m and N (together they yield a_μ), the ghosts b, c , and the counter term δv^2 . Then we go to the vortex sector and set $\phi = \phi_V + \eta$, $A_\mu = A_\mu^V + a_\mu$, but we keep

$v_0^2 = v^2 + \delta v^2$ with the δv^2 which was fixed in the trivial sector. Technically we are using a *minimal renormalization* scheme. Other schemes have been discussed in ^{17,18}.

Note that one cannot require that tadpoles vanish in the soliton sector, because there $\langle \eta \rangle = \phi_1(x)$ is nonvanishing, which plays a role in the calculation of the energy densities¹⁹.

c. The mass correction: The terms in the action which are independent of any quantum fields (η, a_μ, ψ, χ) but which depend on δv^2 lead to a mass counter term ΔM . (Recall that the quantum mass of a soliton is the “vacuum” expectation value of the quantum Hamiltonian). The one-loop quantum correction $M^{(1)}$ to the mass is then given by the Casimir sum

$$M^{(1)} = \pm \left(\sum \frac{1}{2} \hbar \omega_n - \sum \frac{1}{2} \hbar \omega_n^0 \right) + \Delta M \quad (10)$$

where the contributions $\sum \frac{1}{2} \hbar \omega_n^0$ from the trivial sector vanish since we are dealing with a susy system, while $\sum \frac{1}{2} \hbar \omega_n$ sums all discrete and continuous frequencies of the bosonic quantum fluctuations (with a + sign) and fermionic quantum fluctuations (with a - sign).

Working out the linearized field equations for the fluctuations, one finds that the bosonic and fermionic *nonzero* modes have the same solutions in 1-1 correspondence. The zero modes do not contribute to (10). This by itself does not mean that the total sum $\sum \frac{1}{2} \hbar \omega$ vanishes. If one puts the system in a box with boundary conditions on bosons and fermions, one finds in general spurious energy density near the boundaries due to distortion of the various fields⁹. This spurious boundary energy must be subtracted, and then one finds a nonvanishing remainder. One can also work without boundaries, but in the continuum one finds that the difference of spectral densities $\rho_B^{(k)} - \rho_F^{(k)}$ for the bosonic fluctuations and fermionic fluctuations is in general nonzero, and depends on the fields at infinity. (One can also express this difference in terms of a phase shift $\theta'(k)$, but that requires analytical formula for $\theta'(k)$ which is in general not known. We therefore developed an expression²⁰ which only depends on asymptotic values of fields in x -space). However, one finds that for the susy vortex *all nonzero modes cancel*. So, $M^{(1)} = \Delta M$, and since ΔM is linear in δv^2 , and δv^2 is finite but nonzero, one finds a finite but nonzero value for $M^{(1)}$ (see also ²¹).

[The details are as follows (for notation see ¹). The continuous part of the contributions to the energy can be written as

$$\begin{aligned} \langle H_{\text{cont}} \rangle &= \sum \left(\frac{1}{2} \omega_{\text{bos}} - \frac{1}{2} \omega_{\text{ferm}} \right) = \sum (\omega_U - \omega_V) \\ &= \int \frac{d^2 k}{(2\pi)^2} \frac{d^\epsilon \ell}{(2\pi)^\epsilon} \omega_{k,\ell} \int d^2 x [U_{\mathbf{k}}^\dagger U_{\mathbf{k}} - V_{\mathbf{k}}^\dagger V_{\mathbf{k}}](x) \end{aligned} \quad (11)$$

where $\omega_{k,\ell} = (k^2 + \ell^2 + m^2)^{1/2}$ and \mathbf{k} labels the mode functions. Using $V_{\mathbf{k}} = \omega_k^{-1} L U_{\mathbf{k}}$, partially integrating the $V^\dagger V$ term, and using $L^\dagger L U = \omega_k^2 U$,

only a surface integral remains which can be written as

$$\int d^2x (\partial_- F_+ - \partial_+ F_-) = \int d^2x (\partial_x G_y - \partial_y G_x) = \oint d\theta G_\theta \quad (12)$$

where $G_\theta = xG_y - yG_x$ has the following form

$$G_\theta = x^- u_1^* i D_+^\vee u_1 + x^- \sqrt{2} e \phi_V u_1^* u_2 - x^+ u_2^* i \partial_- u_2 - x^+ \sqrt{2} e \phi_V^* u_2^* u_1. \quad (13)$$

Using $x^- \partial_+ = r \partial_r + i \partial_\theta$ and $x^+ \partial_- = r \partial_r - i \partial_\theta$ and $x^- e A_+^\vee \rightarrow in$, we find $\oint d\theta [u_1^* (-\partial_\theta + in) u_1 + u_2^* \partial_\theta u_2] \rightarrow 0$, because the components u_1 and u_2 of U fall off as $r^{-1/2}$ for large r . The terms with ϕ and ϕ^* cancel, and here $N = 2$ susy is at work. Far away u_1 tends to the free mode functions multiplied by $e^{in\theta}$ due to the vortex background, and for free fields the finite (because regularized) expressions $(u_j^* \partial_\theta u_j)$ vanish upon symmetric integration.]

d. The central charge correction: Dimensional reduction of the model in 3+1 dimensions leads in 2+1 dimensions to the following expression for the central charge

$$Z = \int d^2x [\epsilon_{kl} \partial_k \xi_l + P_3]; \quad \xi_l = e(v^2 + \delta v^2) A_l - i \phi^\dagger D_l \phi$$

$$P_3 = F_{0i} F_{3i} + (D_0 \phi)^\dagger D_3 \phi + D_3 \phi^\dagger D_0 \phi - i \bar{\chi} \bar{\sigma}_0 \partial_3 \chi - i \bar{\psi} \bar{\sigma}_0 D_3 \psi \quad (14)$$

Dimensional regularization has brought in a new term not present in 2+1 dimensions: the term P_3 . In the case of the kink such a term yields a nonvanishing contribution to Z which has been interpreted to be a new anomaly. (If the extra dimension shrinks to zero, classically P_3 tends to zero, but at the quantum level $\langle P_3 \rangle$ contains also an infinity due to the summing over infinitely many modes, and “ $0 \times \infty = \text{anomaly}$ ”). But in odd dimensions there are no divergences and thus no anomalies, so one expects P_3 to vanish. Careful study shows it indeed vanishes. But we still need a nonzero connection for Z ! Where is it? The answer is rather simple: most of the loop corrections in ξ_l cancel against the contribution from δv^2 in ξ_l , but there is one term left over:

$$Z = -i \int_0^{2\pi} d\theta \left\langle \eta^\dagger \frac{\partial}{\partial \theta} \eta \right\rangle \Big|_{r=\infty} \quad (15)$$

Asymptotically $|D_k \eta|^2 \rightarrow |\partial_r \eta|^2 + \frac{1}{r^2} |(\partial_\theta - in) \eta|^2$, so η fluctuations have an extra phase $e^{in\theta}$ compared with the trivial vacuum,^b and this topological fact leads to a nonvanishing $Z^{(1)}$ equal to $M^{(1)}$. Saturation holds!

e. Zero modes: It is well-known that differentiation of a classical solution with respect to one of its parameters (such as the coordinates of its center) yields a zero mode of the linearized field equations for the bosonic quantum fluctuations. However, in general this zero mode solution is only a solution

^bOne can also directly compute Z by using the mode expansion $\eta = \sum_{m,k} R_{m,k}(r) e^{-im\theta} e^{in\theta}$.

of the field equations without gauge-fixing term, and not at the same time a solution of that part of the field equations which comes from the gauge fixing term. To obtain a solution of the complete gauge-fixed field equations, one should add a suitable finite gauge transformation (which is always a solution of the non-gauge-fixed field equations). In this way one expects for the vortex two bosonic zero modes, corresponding to the two translations in the x - y plane. A rotation in the x - y plane can be undone by a gauge transformation, so this symmetry does not produce a third bosonic zero mode.

In susy models, one can act with infinitesimal susy transformations on the classical background, and one produces then fermionic zero modes. For the $N = 2$ vortex, one complex susy charge annihilates the bosonic background, while the other complex susy charge generates a complex fermionic zero mode. More in detail: the field equations for the fermionic fluctuations are block-diagonal in two 2-dimensional spaces denoted by $U = \begin{pmatrix} \psi^1 \\ \bar{\chi}^1 \end{pmatrix}$ and $V = \begin{pmatrix} \psi^2 \\ \bar{\chi}^2 \end{pmatrix}$. The (iterated) field equations for U and V read $L^\dagger L U = (\partial_3^2 - \partial_t^2) U$ and $LL^\dagger V = (\partial_3^2 - \partial_t^2) V$, where LL^\dagger is a positive definite operator which has no zero modes, but $L^\dagger L$ has zero modes. To find these zero modes of U one must solve $LU = 0$.

In the literature, it was shown that the iterated (second-order) field equation for ψ^1 has two independent regular solutions, and it was claimed that this meant that there were two independent fermionic zero modes. However, the second solution is *not* a regular solution of the original (first-order) field equation for U (namely $\bar{\chi}^1$ is singular at the origin). Hence there is only one (complex) fermionic zero mode, and this implies that $M = Z$ *must* hold, as we have found.

There is still a remark to be made about the relation between bosonic and fermionic zero modes. A real bosonic zero mode (of the field equations for a_1 , a_2 , $\text{Re } \eta$ and $\text{Im } \eta$) can be written as one complex zero mode for the field equations of $B \equiv \begin{pmatrix} a_1 + ia_2 \\ \eta \end{pmatrix}$. This complex bosonic zero mode is the bosonic partner of the complex fermionic zero mode. Multiplying this pair of complex zero modes by i , the result is of course again a pair of complex zero mode solutions. However, the corresponding *real* solution for a_1 , a_2 , and η , is *linearly independent*: for example, if the first solution corresponds to translations in the x direction, the second one corresponds to translations in the y direction. Similarly, a vortex solution with winding number $n > 1$ has n complex fermionic zero modes and $2n$ real bosonic zero modes.

We conclude: there is only one (complex) fermionic zero mode (corresponding to one pair of fermionic annihilation and creation operators), and this gives rise to a single short multiplet at the quantum level (a massive multiplet as short as a massless multiplet). Standard multiplet shortening arguments²² therefore do apply and explain the preservation of the Bogomolnyi saturation that we verified by explicit calculation at the one-loop level. An enormous simplification in these calculations was the use of dimensional

regularization which does not need any boundary conditions.

References

1. A. Rebhan, P. van Nieuwenhuizen and R. Wimmer, *Nucl. Phys.* **B679**, 382 (2004).
2. R. Rajaraman, *Solitons and Instantons* (North-Holland, Amsterdam, 1982).
3. J. F. Schonfeld, *Nucl. Phys.* **B161**, 125 (1979).
4. R. K. Kaul and R. Rajaraman, *Phys. Lett.* **B131**, 357 (1983).
5. C. Imbimbo and S. Mukhi, *Nucl. Phys.* **B247**, 471 (1984).
6. A. Rebhan and P. van Nieuwenhuizen, *Nucl. Phys.* **B508**, 449 (1997).
7. H. Nastase, M. Stephanov, P. van Nieuwenhuizen and A. Rebhan, *Nucl. Phys.* **B542**, 471 (1999).
8. N. Graham and R. L. Jaffe, *Nucl. Phys.* **B544**, 432 (1999).
9. M. A. Shifman, A. I. Vainshtein and M. B. Voloshin, *Phys. Rev.* **D59**, 045016 (1999).
10. E. B. Bogomolnyi, *Sov. J. Nucl. Phys.* **24**, 449 (1976).
11. J. R. Schmidt, *Phys. Rev.* **D46**, 1839 (1992).
12. J. Edelstein, C. Núñez and F. Schaposnik, *Phys. Lett.* **B329**, 39 (1994).
13. B.-H. Lee and H. Min, *Phys. Rev.* **D51**, 4458 (1995).
14. B.-H. Lee, C.-k. Lee and H. Min, *Phys. Rev.* **D45**, 4588 (1992).
15. R. K. Kaul, *Phys. Lett.* **B143**, 427 (1984).
16. C. Imbimbo and S. Mukhi, *Nucl. Phys.* **B249**, 143 (1985).
17. A. Rebhan, P. van Nieuwenhuizen and R. Wimmer, *New J. Phys.* **4**, 31 (2002).
18. A. Rebhan, P. van Nieuwenhuizen and R. Wimmer, *Nucl. Phys.* **B648**, 174 (2003).
19. A. S. Goldhaber, A. Litvintsev and P. van Nieuwenhuizen, *Phys. Rev.* **D67**, 105021 (2003).
20. A. S. Goldhaber, A. Rebhan, P. van Nieuwenhuizen and R. Wimmer, “Quantum corrections to mass and central charge of supersymmetric solitons”, contribution to the Hidenaga Yamagishi commemorative volume of Physics Reports, edited by E. Witten and I. Zahed.
21. D. V. Vassilevich, *Phys. Rev.* **D68**, 045005 (2003).
22. E. Witten and D. Olive, *Phys. Lett.* **B78**, 97 (1978).